

## $\beta^*$ - Locally Closed Sets in Topological Spaces

K. Rajendra Suba  
PG Department of Mathematics  
A.P.C. Mahalaxmi College for Women, Thoothukudi

### Abstract

In this paper we introduce three forms of  $\beta^*$ - locally closed sets called  $\beta^*$ - LC sets,  $\beta^*$ - LC\* sets and  $\beta^*$ - LC\*\* sets. Properties of these new concepts are studied as well as their relations to the other classes of locally closed sets will be investigated. Additionally, we define  $\beta^*$ - Locally continuous function and compare it with other locally continuous functions in topological spaces.

**Keywords:**  $\beta^*$ - LC sets,  $\beta^*$ - LC\* sets and  $\beta^*$ - LC\*\* sets.

### I. Introduction

The notion of a locally closed set in a topological space was introduced by kurutowski and seerpinski [10]. According to bourbaki [3] a subset A of a topological space X is called locally closed in X if it is the intersection of an open set and a closed set in X. Ganster and Reilly used locally closed sets to define LC - continuity and LC - irresoluteness. Balachandran, Sundaram and Maki [2] introduced the concept of generalized locally closed sets in topological spaces and introduced some of their properties. Also various authors like, Arockiarani, Gnanambal, Park and Park and Veera Kumar [8] have introduced regular-generalized locally closed sets, semi-generalized locally closed sets and  $g^*$ - locally closed sets respectively in topological spaces.

In this paper, we introduce three weaker forms of locally closed sets denoted by  $\beta^*$ - LC(X,  $\tau$ ),  $\beta^*$ - LC\*(X,  $\tau$ ),  $\beta^*$ - LC\*\*(X,  $\tau$ ) each of which contains LC(X,  $\tau$ ) and obtained some of their properties and also their relationships with other locally closed sets.

### II. Preliminaries

**Definition 2.1:** A subset A of a topological space (X,  $\tau$ ) is called locally closed [3], if  $A = U \cap F$ . where  $U \in \tau$  and F is closed in (X,  $\tau$ ).

**Definition 2.2:** A subset A of a topological space (X,  $\tau$ ) is called  $\beta$  - locally closed ( briefly  $\beta$  - LC ) set if  $A = U \cap V$ . Where U is  $\beta$  - open and V is  $\beta$  - closed.

**Definition 2.3:** A subset S of (X,  $\tau$ ) is called g - locally closed set[2] (briefly g- lc) if  $S = G \cap F$ . where G is g - open in (X,  $\tau$ ) and F is g - closed in (X,  $\tau$ ) . Every g - closed set (resp. g - open set) is g- lc.

**Definition 2.4:** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called  $\beta$  - continuous if  $f^{-1}(O)$  is a  $\beta$  - open of (X,  $\tau$ ) for every open set o of (Y,  $\sigma$ ).

**Definition 2.5:** A subset  $A$  of a topological space is said to be  $\beta^*$ - open if  $A \subseteq \text{cl}(\text{int}^*(\text{cl}(A)))$ .

**Definition 2.6:** The complement of  $\beta^*$ - open set in  $X$  is  $\beta^*$  - closed set in  $X$ .

### III. $\beta^*$ - Locally Closed Sets

In this section we introduce three forms of locally closed sets denoted by  $\beta^*$ -locally closed sets,  $\beta^*$ -  $LC^*$  sets and  $\beta^*$ -  $LC^{**}$  sets and obtain some of their properties and also their relationships with  $g$ - lc sets,  $\beta$  - locally closed sets.

**Definition 3.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called a  $\beta^*$ - locally closed set if  $A = S \cap F$ . Where  $S$  is  $\beta^*$ - open and  $F$  is  $\beta^*$  - closed.

The class of all  $\beta^*$ - locally closed sets in  $(X, \tau)$  is denoted by  $\beta^*$ -  $LC(X, \tau)$ .

**Example 3.2:** Let  $X = \{ a, b, c \}$ ,  $\tau = \{\phi, \{a, b\}, X\}$ ,  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$  and  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$ .

Then  $\beta^*$ -  $LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}, X\}$ .

**Definitions 3.3:** A subset  $A$  of a topological space  $(X, \tau)$  is said to be  $\beta^*$ -  $LC^*$  set if there exists  $\beta^*$ - open set  $S$  and a closed set  $F$  of  $(X, \tau)$  such that  $A = S \cap F$ .

**Example 3.4:** Let  $X = \{ a, b, c, d \}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ ,  $\tau^c = \{\phi, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*$ -  $LC^*(X, \tau) = \{\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .

**Definitions 3.5:** A subset  $A$  of a topological space  $(X, \tau)$  is said to be  $\beta^*$ -  $LC^{**}$  - set if there is an open set  $S$  and a  $\beta^*$ - closed set  $F$  of  $(X, \tau)$  such that  $A = S \cap F$ .

**Example 3.6:** Let  $X = \{ a, b, c, d \}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$ ,  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*$ -  $LC^{**}(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ .

**Theorem 3.7:** If a subset  $A$  of  $(X, \tau)$  is locally closed then it is  $\beta^*$ -  $LC(X, \tau)$ ,  $\beta^*$ -  $LC^*(X, \tau)$ ,  $\beta^*$ -  $LC^{**}(X, \tau)$  set.

**Proof :** Let  $A = P \cap Q$  Where  $P$  is open and  $Q$  is closed in  $(X, \tau)$ . Since every open set is  $\beta^*$ -open and every closed is  $\beta^*$ - closed,  $A$  is  $\beta^*$ -  $LC(X, \tau)$ ,  $\beta^*$ -  $LC^*(X, \tau)$ ,  $\beta^*$ -  $LC^{**}(X, \tau)$ .

**Remark 3.8 :** The Converse of the above theorem need not be true as seen from following example.

**Example 3.9:** Let  $X = \{ a, b, c, d \}$ ,  $\tau = \{\phi, \{a\}, \{b, c, d\}, X\}$ ,  $\tau^c = \{\phi, \{a\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*$ -  $LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$  and  $LC(X, \tau) = \{\phi, \{a\}, \{b, c, d\}, X\}$ . Here  $\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}$ ,

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}$  is  $\beta^*$ -LC  $(X, \tau)$  but not LC  $(X, \tau)$ .

**Example 3.10:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$ ,  $\tau^c = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$  Then  $\beta^*-LC^*(X, \tau) = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$  and  $LC(X, \tau) = \{\phi, \{c\}, \{a, b\}, X\}$ . Here  $\{a\}, \{a, c\}$  is  $\beta^*-LC^*(X, \tau)$  but not LC  $(X, \tau)$ .

**Example 3.11:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$ ,  $\tau^c = \{\phi, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*-LC^{**}(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, d\}, X\}$  and  $LC(X, \tau) = \{\phi, \{c\}, \{a, b, d\}, X\}$ . Here  $\{a\}, \{b\}, \{a, c\}, \{b, c\}$  is  $\beta^*-LC^{**}(X, \tau)$  but not LC  $(X, \tau)$ .

**Theorem 3.12:** If a subset  $A$  of  $(X, \tau)$  is  $\beta^*-LC^*$ -set then it is  $\beta^*-LC$ -set.

**Proof:** Let  $A$  be a  $\beta^*-LC^*$ -set. Let  $P$  be a  $\beta^*$ -set in  $(X, \tau)$  and  $Q$  be closed set in  $(X, \tau)$ . Since  $A$  is  $\beta^*-LC^*$ -set by definitions,  $A = P \cap Q$ . Since every closed set is  $\beta^*$ -closed then  $Q$  is  $\beta^*$ -closed. Then  $A$  is  $\beta^*-LC$ -set.

**Remark 3.13:** The converse of the above theorem need not be true as seen from the following example.

**Example 3.14:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ ,  $\tau^c = \{\phi, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*-LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$  and  $\beta^*-LC^*(X, \tau) = \{\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Here  $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, d\}$  is  $\beta^*-LC(X, \tau)$  but not  $\beta^*-LC^*(X, \tau)$ .

**Theorem 3.15:** Every  $\beta^*-LC^{**}(X, \tau)$  is  $\beta^*-LC(X, \tau)$ .

**Proof:** It follows from the definitions 3.1 and 3.3.

**Remark 3.16:** The converse of the above theorem need not be true as seen from the following example.

**Example 3.17 :** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$ ,  $\tau^c = \{\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*-LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$  and  $\beta^*-LC^{**}(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ . Here  $\{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$  is  $\beta^*-LC(X, \tau)$  but not

$\beta^*$ -  $LC^{**}(X, \tau)$ .

**Theorem 3.18:** If a subset  $A$  of  $(X, \tau)$  is  $\beta$  -  $LC$  set then  $A$  is  $\beta^*$ -  $LC$  - set.

**Proof:** Let  $A = P \cap Q$ . Where  $P$  is  $\beta$  - Open,  $Q$  is  $\beta$  - Closed in  $(X, \tau)$ . Since, every  $\beta$  - Open set is  $\beta^*$ - Open and every  $\beta$  - Closed set is  $\beta^*$ - Closed. Therefore,  $A$  is  $\beta^*$ -  $LC$  - set in  $(X, \tau)$ .

**Remark 3.19:** The Converse of the above theorem need not be true as seen from the following example.

**Example 3.20:** Let  $X = \{ a, b, c, d \}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ ,  $\tau^c = \{\phi, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta O(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ ,  $\beta C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ , Then  $\beta^*$ -  $LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$  and  $\beta$  -  $LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Here  $\{c\}, \{a, b\}, \{a, d\}, \{c, d\}, \{a, b, d\}$  is  $\beta^*$ -  $LC(X, \tau)$  but not  $\beta$  -  $LC(X, \tau)$ .

**Remark 3.21:** Intersection of two  $\beta^*$ -  $LC$  sets (resp,  $\beta^*$ -  $LC^*$  - sets,  $\beta^*$ -  $LC^{**}$  -sets) need not be a  $\beta^*$ -  $LC$  (resp,  $\beta^*$ -  $LC^*$  - sets,  $\beta^*$ -  $LC^{**}$  - sets) as seen from the following example.

**Example 3.21:** Let  $X = \{ a, b, c, d \}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$ ,  $\tau^c = \{\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $\beta^*$ -  $LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Here  $\{a, d\}, \{b, c, d\}$  is in  $\beta^*$ -  $LC(X, \tau)$  but  $\{a, d\} \cap \{b, c, d\} = \{d\}$  is not in  $\beta$  -  $LC(X, \tau)$ . Also  $\beta^*$ -  $LC^*(X, \tau) = \{\phi, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Here  $\{a, d\}, \{b, d\}$  is in  $\beta^*$ -  $LC^*(X, \tau)$  but  $\{a, d\} \cap \{b, d\} = \{d\}$  is not in  $\beta^*$  -  $LC^*(X, \tau)$ .

#### IV. $\beta^*$ - $LC$ Continuous Function in Topological Space

In this section we introduce the concept of  $\beta^*$ -  $LC$  continuous and  $\beta^*$ -  $LC^*$  Continuous and  $\beta^*$ -  $LC^{**}$  Continuous functions maps are defined and some of their properties are obtained.

**Definition 4.1:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a map. Then  $f$  is called

- (i)  $\beta^*$ -  $LC$  continuous if  $f^{-1}(V) \in \beta^*$ -  $LC(X, \tau)$  for each  $V \in (Y, \sigma)$
- (ii)  $\beta^*$ -  $LC^*$  continuous if  $f^{-1}(V) \in \beta^*$ -  $LC^*(X, \tau)$  for each  $V \in (Y, \sigma)$
- (iii)  $\beta^*$ -  $LC^{**}$  continuous if  $f^{-1}(V) \in \beta^*$ -  $LC^{**}(X, \tau)$  for each  $V \in (Y, \sigma)$

**Example 4.2 :** Let  $X = Y = \{ a, b, c \}$ ,  $\tau = \{\phi, \{a, b\}, X\}$ ,  $\tau^c = \{\phi, \{c\}, X\}$ ,  $\sigma = \{\phi, \{a\}, Y\}$ ,  $\sigma^c = \{\phi, \{b, c\}, Y\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ .

$\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$  Then  $\beta^*LC(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, c\}, \{b, c\}, X\}$ . Then the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  defined by  $f(a) = b, f(b) = c, f(c) = a$ . Clearly  $f$  is  $\beta^*LC$  continuous.

**Example 4.3 :** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, \{c\}, \{a, b\}, X\}$ ,  $\tau^c = \{\phi, \{c\}, \{a, b\}, X\}$ ,  $\sigma = \{\phi, \{a, b\}, Y\}$ ,  $\sigma^c = \{\phi, \{c\}, Y\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ .

$\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$  Then  $\beta^*LC(X, \tau) = \{\phi, \{c\}, \{a, b\}, X\}$ . Then the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  defined by  $f(a) = a, f(b) = b, f(c) = c$ . Clearly  $f$  is  $\beta^*LC^*$  continuous.

**Example 4.4 :** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$ ,  $\tau^c = \{\phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ ,  $\sigma = \{\phi, \{a, b\}, Y\}$ ,  $\sigma^c = \{\phi, \{c, d\}, Y\}$ .  $\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ .  $\beta^*C(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$  Then  $\beta^*LC^{**}(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ . Then the function  $f : (X, \tau) \rightarrow (Y, \sigma)$  defined by  $f(a) = b, f(b) = c, f(c) = a$ . Clearly  $f$  is  $\beta^*LC^{**}$  continuous.

**Theorem 4.5:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a function. Then we have the following

1. If  $f$  is LC - continuous then  $f$  is  $\beta^*LC$  continuous,  $\beta^*LC^*$  continuous,  $\beta^*LC^{**}$  continuous.
2. If  $f$  is  $\beta^*LC^*$  continuous (or)  $\beta^*LC^{**}$  continuous function the  $f$  is  $\beta^*LC$  continuous.

**Proof:**

1. Suppose that  $f$  is LC – continuous. Let  $V$  be an open set of  $(X, \tau)$ . Then  $f^{-1}(V)$  is locally closed in  $(X, \tau)$ . Since every locally closed set is  $\beta^*LC$  set,  $\beta^*LC^*$  set,  $\beta^*LC^{**}$  set, it follows that  $f$  is  $\beta^*LC$  continuous,  $\beta^*LC^*$  continuous,  $\beta^*LC^{**}$  continuous.

2. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a  $\beta^*LC^*$  continuous (or)  $\beta^*LC^{**}$  continuous function.

Since every  $\beta^*LC^*$  set is  $\beta^*LC$  set and every  $\beta^*LC^{**}$  set is  $\beta^*LC$  set. Therefore the proof follows.

**Remark 4.6:** The converse of the above theorem need not be true as seen from the following example.

**Example 4.7:**

**1) Every  $\beta^*LC$  continuous is not LC continuous:**

Let  $X = \{a, b, c, d\}, Y = \{a, b, c, d\}$   $\tau = \{X, \phi, \{a\}, \{b, c, d\}\}$  &  $\sigma = \{X, \phi, \{a\}, \{a, b, c\}\}$ ,  $\beta^*LC(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ . Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  define by  $f(a) = a, f(b) = b, f(c) = c,$

$f(d) = d$ . Clearly  $f$  is  $\beta^*$ - LC continuous but not locally closed, since  $f^{-1} \{a, b, c\} = abc$  is not LC in  $X$ .

## 2) Every $\beta^*$ - $LC^*$ continuous is not LC continuous:

Let  $X = \{a, b, c\}$ ,  $Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$  &  $\sigma = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ ,  $\beta^*$ -  $LC^*$  ( $X, \tau$ ) =  $\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ . Let  $f: X \rightarrow Y$  defined by  $f(a) = a$ ,  $f(b) = b$ ,  $f(c) = c$ . Clearly  $f$  is  $\beta^*$ -  $LC^*$  continuous but  $f^{-1} \{a, c\} = ac$  is not LC in  $X$ . Hence  $f$  is not LC continuous in  $X$ .

## 3) Every $\beta^*$ - $LC^{**}$ continuous is not LC - continuous:

Let  $X = \{a, b, c\}$ ,  $Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$  &  $\sigma = \{X, \phi, \{a, b\}\}$ ,  $\beta^*$ -  $LC^{**}$  ( $X, \tau$ ) =  $\{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ . Let  $f: X \rightarrow Y$  defined by  $f(a) = c$ ,  $f(b) = a$ ,  $f(c) = b$ . Clearly  $f$  is  $\beta^*$ -  $LC^{**}$  continuous but  $f^{-1} \{a, b\} = bc$  is not LC in  $X$ . Hence  $f$  is not LC continuous in  $X$ .

## 4) Every $\beta^*$ - LC continuous is not $\beta^*$ - $LC^*$ continuous:

Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$   $\sigma = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$   $\beta^*$ - LC ( $X, \tau$ ) =  $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ ,  $\beta^*$ -  $LC^*$  ( $X, \tau$ ) =  $\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ . Let  $f: X \rightarrow Y$   $f(a) = a$ ,  $f(b) = b$ ,  $f(c) = c$ . Clearly  $f$  is  $\beta^*$ - LC continuous but not  $\beta^*$ -  $LC^*$  continuous since  $f^{-1} \{b, c\} = \{b, c\}$  is not in  $\beta^*$ -  $LC^*$ .

## 5) Every $\beta^*$ - LC continuous is not $\beta^*$ - $LC^{**}$ continuous:

Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$ .  $\beta^*$ - LC ( $X, \tau$ ) =  $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{b, c, d\},$

$\{a, b, d\}, \{a, c, d\}\}$ .  $\beta^*$ -  $LC^{**}$  ( $X, \tau$ ) =  $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, d\}\}$ .

$\sigma = \{X, \phi, \{a\}, \{a, b, c\}\}$ , Let  $f: X \rightarrow Y$   $f(a) = a$ ,  $f(b) = d$ ,  $f(c) = b$ ,  $f(d) = c$ . Clearly  $f$  is

$\beta^*$ - LC continuous but not  $\beta^*$ -  $LC^{**}$ .

**Theorem 4.8:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $\beta^*$ - LC continuous and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  is continuous, then  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is  $\beta^*$ - LC continuous function.

**Proof:** Let  $F$  be a closed set in  $(Z, \eta)$ . Then  $g^{-1}(F)$  is closed in  $(Y, \sigma)$ . Since  $g$  is continuous, then  $f^{-1}(g^{-1}(F))$  is  $\beta^*$ - LC set in  $(X, \tau)$  as  $f$  is  $\beta^*$ - LC continuous then  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is  $\beta^*$ - LC continuous function.

**Theorem 4.9 :** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be any two functions. Then

- (i)  $g \circ f$  is  $\beta^*$ -LC<sup>\*</sup>-continuous if  $f$  is  $\beta^*$ -LC<sup>\*</sup>-continuous and  $g$  is continuous.
- (ii)  $g \circ f$  is  $\beta^*$ -LC<sup>\*\*</sup>-continuous if  $f$  is  $\beta^*$ -LC<sup>\*\*</sup>-continuous and  $g$  is continuous.

**Proof:** The proof is similar to that of Theorem 4.8.

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